Y_1^*(1385 MeV): Study of Spin and Parity by Moment Analysis for $J=5/2$, $3/2$, and $1/2$*

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The possibility that the Y_1^*(1385-MeV) spin is $\frac{5}{2}$ has been investigated. Identification of the Y_1^* as an $F_{5/2}$ state is excluded; the $D_{5/2}$ assignment is not required; and $J=\frac{5}{2}$ is found rather unlikely. Of the states $P_{3/2}$, $D_{3/2}$, $S_{1/2}$, and $P_{1/2}$, only $P_{3/2}$ is acceptable, the excluded hypotheses having confidence levels of order 10^{-6} or 10^{-7}; this conclusion strengthens slightly the authors' earlier selection of $P_{3/2}$, based chiefly on study of the θ_Λ dependence of two components of the decay Λ's polarization. This more definitive report results from the application of the "moment analysis" of Byers and Fenster to a sample of 895 Y_1^* decays obtained from $K^- - p$ interactions in the 72-in. bubble chamber.

I. INTRODUCTION

The spin and parity of the Y_1^* (1385 MeV) has been investigated for $J=\frac{5}{2}$, $\frac{3}{2}$, and $\frac{1}{2}$. The identification of the Y_1^* as an $F_{5/2}$ state is excluded; the $D_{5/2}$ assignment is not required by the data. Direct evaluation of $2J+1$ indicates $J=\frac{3}{2}$ to be more probable than $J=\frac{5}{2}$. Confirmatory evidence has been obtained for the authors' earlier selection of $P_{3/2}$, based chiefly on the angular dependence of the decay Λ's polarization.[1]

A number of reported experiments show considerable anisotropy in the angular distribution of $Y_1^{*\pm} \to \Lambda+\pi^\pm$ decay and thus permit the conclusion that the Y_1^* has a spin $>\frac{1}{2}$.[2] Two experimental groups have found that the average polarization of the Λ indicates that the Y_1^* is either $S_{1/2}$ or $P_{3/2}$.[3] A recent study of $Y_1^{*0} \to \Lambda+\pi^0$ events concludes from an Adair distribution that spin $\frac{3}{2}$ is probable but that spin $\frac{5}{2}$ is unlikely.[4,5]

The analysis reported here utilizes the complete angular distribution of all three components of Λ polarization (whereas the earlier study treated only the ϕ_Λ-averaged distributions of the polarization components in the plane of the Λ and the production normal). The parity conclusion is based on a comparison of the polarization components transverse and parallel to the Λ direction; this comparison is similar to (but is more general than) that of the normal and "magic-direction" components of polarization made earlier. In this present study, the formalism of Byers and Fenster[6] is applied to the distributions of Λ direction and Λ polarization resulting from Y_1^* decay.

II. THEORY

As Byers and Fenster have shown, the decay process $Y^* \to \Lambda+\pi$ can be conveniently described in terms of expectation values t_L^M of certain spin-space operators[7] T_L^M; these expectation values are designated as "multipole parameters." Thus the angular distribution of the decay Λ is given by

$$I(\theta,\phi)=\sum_{L,M} n_{L0} t_L^M Y_L^{M*}(\theta,\phi) \qquad (1)$$

with L even,[8] where n_{L0} is a constant determined by L and J (the Y_1^* spin); $Y_L^{M*}(\theta,\phi)$ is the charge conjugate of the usual spherical harmonic; and the normalization is such that $\int I(\theta,\phi)d\Omega=1$. The angular dependence of the Λ polarization is represented by

$$IP_1^m(\theta,\phi)=\sum_{L',M'} G_{L'M'}^{(m)} Y_{L'}^{M'*}(\theta,\phi)\,, \qquad (2)$$

with L' even, where P_1^m represents a spherical tensor polarization component [$P_1^0=P_z$, $P_1^1=-(P_x+iP_y)/\sqrt{2}$, and $P_1^{-1}=(P_x-iP_y)/\sqrt{2}$]; and $G_{L'M'}^{(m)}$ is a combina-

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[1] J. B. Shafer, J. J. Murray, and D. O. Huwe, Phys. Rev. Letters **10**, 179 (1963).

[2] R. P. Ely, S. Y. Fung, G. Gidal, Y. L. Pan, W. M. Powell, and H. S. White, Phys. Rev. Letters **7**, 461 (1961); L. Bertanza, V. Brisson, P. L. Connolly, E. L. Hart, I. S. Mittra, G. C. Moneti, R. R. Rau, N. P. Samios, I. O. Skillicorn, S. S. Yamamoto, M. Goldberg, J. Leitner, S. Lichtman, and J. Westgard, *ibid.* **10**, 176 (1963).

[3] A. R. Erwin, R. H. March, and W. D. Walker, Nuovo Cimento **24**, 237 (1962); D. Colley, N. Gelfand, U. Nauenberg, J. Steinberger, S. Wolf, H. R. Brugger, P. R. Kramer, and R. J. Plano, in *Proceedings of the International Conference on High-Energy Nuclear Physics* (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 315.

[4] L. J. Curtis, C. T. Coffin, D. J. Meyer, and K. M. Terwilliger, Phys. Rev. **132**, 1771 (1963). The assignment of spin $\frac{3}{2}$ requires the assumption of S and P waves in production.

[5] Robert K. Adair, Yale University (unpublished) has recently attempted to reproduce the results of several experiments by assuming special forms of backgrounds with the production of spin-$\frac{1}{2}$ Y_1^*'s. We believe that the small background in our experiment (probably <5%, certainly <10%) requires such special assumptions to account for observed high-order moments as to make the hypothesis unlikely. The three experiments cited by Adair to support spin $\frac{1}{2}$ are somewhat limited by statistics and background; two of these have been simply interpreted by the experimenters to support spin $\frac{3}{2}$ (Refs. 3 and 4).

[6] N. Byers and S. Fenster, Phys. Rev. Letters **11**, 52 (1963).

[7] These T_L^M operators are the irreducible tensors defined in M. E. Rose, *Elementary Theory of Angular Momentum* (John Wiley & Sons, Inc., New York, 1957); or A. R. Edmonds, *Angular Momentum in Quantum Mechanics* (Princeton University Press, Princeton, New Jersey, 1957). The normalization of each T_L^M, as in Byers and Fenster, is such that the density matrix for the Y_1^* spin states is $\rho=(2J+1)^{-1}\Sigma_{L,M}(2L+1)t_L^{M*}T_L^M$.

[8] These expressions result from the requirement that the angular distribution be a scalar quantity (as it is specified by one number for each θ, ϕ); and that the polarization be a vector quantity, i.e., a tensor of rank one. The T_L^M's have the same form in spin space that the Y_L^M's have in coordinate space; e.g., $T_1^1=-(S_x+iS_y)/[2J(J+1)]^{1/2}$. For use of the T_L^M's in collision problems, see the theoretical study of W. Lakin, Phys. Rev. **98**, 139 (1955), and the experimental work of J. Button and R. Mermod, *ibid.* **118**, 1333 (1960), on vector and tensor polarizations of deuterons.

tion of $t_{L'+1}{}^{m+M'}$ and $t_{L'-1}{}^{m+M'}$.[8,9] The angles θ and ϕ are the polar and azimuthal angles of the Λ momentum as observed in the Y_1* rest frame (in a coordinate system determined by the production process); the z axis is most conveniently taken as the normal (**n**) to the production plane, as all $t_L{}^M$ with M odd must then vanish (by reason of parity conservation in production).

For the determination of the Y_1* parity, Byers and Fenster transform the above components of $I\bar{P}(\theta,\phi)$ into components longitudinal and transverse with respect to the Λ direction. Both of these involve only the odd-L $t_L{}^M$ as before, though the dependence on spherical harmonics is very different. The longitudinal component has the form

$$IP_{\parallel}=I\bar{P}\cdot\hat{\Lambda}=\sum_{L,M} n_{L0}t_L{}^M Y_L{}^{M*}(\theta,\phi) \qquad (3)$$

with L odd; and the transverse component can be represented as

$$\begin{aligned}IP_{\perp}&=I\bar{P}\cdot\mathbf{\Lambda}\times(\mathbf{\Lambda}\times\mathbf{n})/|\mathbf{\Lambda}\times(\mathbf{\Lambda}\times\mathbf{n})|\\&\quad-iI\bar{P}\cdot(\mathbf{n}\times\mathbf{\Lambda})/|\mathbf{n}\times\mathbf{\Lambda}|\\&=-\sum_{L,M} n_{L1}[(2L+1)/4\pi]^{1/2}\mathfrak{D}_{M1}{}^{L*}(\phi,\theta,0)\gamma t_L{}^{M*} \qquad (4)\end{aligned}$$

with L odd, where n_{L1} depends on L and J, $\mathfrak{D}_{M1}{}^L$ is the usual rotation operator, and $\gamma=+1$ or -1 in accordance with the Y_1* parity ($J=l+\frac{1}{2}$ or $l-\frac{1}{2}$).

The $t_L{}^M$ multipole parameters are proportional to the "moments" $\langle Y_L{}^M\rangle$ of the $I(\theta,\phi)$ and $IP(\theta,\phi)$ distributions (and will hereafter be referred to as "moments"). These can be evaluated by multiplying each distribution by the appropriate $Y_L{}^M$ (or by $\mathfrak{D}_{M1}{}^L$ for the transverse polarization) and integrating. Thus from the angular distribution,

$$n_{L0}t_L{}^M=\int d\Omega I(\theta,\phi)Y_L{}^M(\theta,\phi) \qquad (5)$$

for even L; and from the longitudinal polarization,

$$n_{L0}t_L{}^M=\int d\Omega I\bar{P}\cdot\hat{\Lambda}(\theta,\phi)Y_L{}^M(\theta,\phi) \qquad (6)$$

for odd L. Finally, from the transverse polarization,

$$\gamma n_{L1}t_L{}^{M*}=[(2L+1)/4\pi]^{1/2}\times\int d\Omega\,\mathfrak{D}_{M1}{}^L(\phi,\theta,0)IP_{\perp}(\theta,\phi) \qquad (7)$$

for odd L. Since

$$P_{\perp}=-\sqrt{2}\sum_m \mathfrak{D}_{m1}{}^{1*}(\phi,\theta,0)P_1{}^{m*},$$

Eq. (7) reduces to

$$\gamma n_{L1}t_L{}^M=(2L+1)^{-1/2}\left[\sum_m A_L\int d\Omega IP_1{}^m Y_{L-1}{}^{M-m}+\sum_m B_L\int d\Omega IP_1{}^m Y_{L+1}{}^{M-m}\right], \qquad (8)$$

where m has values 0, $+1$, or -1; $A_L=(L+1)^{1/2}\,C(1,L-1,L;m,M-m)$ and $B_L=(L)^{1/2}\,C(1,L+1,L;m,M-m)$; and $P_1{}^m$ represents a spherical tensor polarization component. Comparison of the quantities evaluated from expressions (6) and (8), as shown by Byers and Fenster, determines γ and hence the Y_1* parity.

III. APPLICATION

In practice, an angular distribution moment is determined by evaluating the appropriate $Y_L{}^M$ for the θ, ϕ angles of each event (k) and summing over the N events to find the average[10]:

$$n_{L0}t_L{}^M=\left[\sum_{k=1}^N Y_L{}^M(\theta_k,\phi_k)\right](1/N). \qquad (9)$$

[Compare with Eq. (5).] Polarization moments are more difficult to evaluate because the polarization itself is an average, found by summing direction cosines of decay pion momenta in the Λ rest frame; thus

$$\bar{P}\cdot\hat{\Lambda}(\theta,\phi)=(3/\alpha n)\sum_{j=1}^n \hat{\pi}_j\cdot\hat{\Lambda}(\theta,\phi),$$

or (10)

$$P_1{}^m(\theta,\phi)=(3/\alpha n)\sum_{j=1}^n(4\pi/3)^{1/2}Y_1{}^m(\Theta_j,\Phi_j),$$

where α is the Λ decay parameter[10a] ($\alpha=-0.62$), j is any event with a Λ traveling in the θ, ϕ direction in the Y* rest frame, $\hat{\pi}$ is a unit vector along the pion direction in the Λ rest frame, and $\hat{\Lambda}$ is along the direction of transformation into this rest frame. Angles Θ_j, Φ_j refer to the $\hat{\pi}$ direction in the Λ rest frame.[10] Equation (6) for the odd-L moments of longitudinal polarization becomes [with absorption of the sum of Eq. (10) into that of

[9] "Polarization" will always refer to $I\bar{P}(\theta,\phi)$, the fractional number of decay Λ's times the Λ polarization for some θ, ϕ direction.

[10] Particle four-momenta are Lorentz-transformed from the laboratory system to the production c.m., then to the Y_1* rest frame, and finally (Λ decay products) to the Λ rest frame. The θ, ϕ angles for the $\hat{\Lambda}$ and the Θ, Φ angles for the $\hat{\pi}$ are determined by taking appropriate direction cosines with respect to axes obtained by "direct Lorentz transformations" of x, y, and z axes (incident $\hat{k}\times\hat{n}$, $\hat{k}$, and $\hat{n}$ directions) to each successive system. The direct Lorentz "transformation" is simply the orienting of x, y, and z axes the same in the new system as in the old, with respect to the β direction of the conventional Lorentz transformation. [See H. P. Stapp, Lawrence Radiation Laboratory Report, UCRL-8096, 1957 (unpublished).]

[10a] *Note added in proof.* This α is defined with sign opposite to that of the α which has now become conventional. See J. W. Cronin and O. E. Overseth, Phys. Rev. **129**, 1795 (1963).

Eq. (6)]:

$$n_{L0}t_L^M=[\sum_{k=1}^{N} Y_L^M(\theta_k,\phi_k)\hat{\pi}_k\cdot\hat{\Lambda}_k](3/\alpha N). \quad (11)$$

Equation (8) for the odd-L moments of transverse polarization becomes

$$\gamma n_{L1}t_L^M=(2L+1)^{-1/2} \times[\sum_m A_L \sum_{k=1}^{N} Y_{L-1}^{M-m}(\theta_k,\phi_k)Y_1^m(\Theta_k,\Phi_k) +\sum_m B_L \sum_{k=1}^{N} Y_{L+1}^{M-m}(\theta_k,\phi_k)Y_1^m(\Theta_k,\Phi_k)] \times(3/\alpha N)(4\pi/3)^{1/2}. \quad (12)$$

The expressions above obviously are sums of complex numbers, so represent separate sums of real and imaginary parts.

Errors are evaluated for the real and imaginary parts of each moment by the use of such expressions as

$$\delta(\mathrm{Re}t_L^M)=(1/n_{L0})\{\sum_{k=1}^{N}[\mathrm{Re}Y_L^M(\theta_k,\phi_k)]^2 -[\sum_{k=1}^{N}\mathrm{Re}Y_L^M(\theta_k,\phi_k)]^2/N\}^{1/2}(1/N) \quad (13)$$

for the real part of the cross-section moment found from Eq. (9), and

$$\delta(\mathrm{Re}t_L^M)=(1/n_{L0})\{\sum_{k=1}^{N}(\mathrm{Re}Y_L^M)^2(\hat{\pi}\cdot\hat{\Lambda})^2 -[\sum_{k=1}^{N}\mathrm{Re}Y_L^M\hat{\pi}\cdot\hat{\Lambda}]^2/N\}^{1/2}(3/\alpha N) \quad (14)$$

for the polarization moment of Eq. (11). (The second term usually is very small in comparison with the first in these equations.)

IV. EXPERIMENT

The formulas developed above were applied to the Y_1^* decay distributions from 895 specially selected events from a sample of 1650 identified as

$$K^-+p\rightarrow Y_1^{*\pm}+\pi^{\mp} \quad (15)$$

at 1.22 BeV/c incident momentum. The selection criteria were that the Y_1^* mass be between 1340 and 1430 MeV and that the production angle be such that $|\hat{Y}^*\cdot\hat{K}|\leq 0.8$. The mass limits gave good separation between Y^{*+} and Y^{*-} bands on the Dalitz plot, shown in Fig. 1; the restriction on production angle enhanced the observed polarization or alignment. (For other details, see the earlier reports.[11])

[11] Preliminary results are given by J. Button-Shafer, D. Huwe, and J. J. Murray in *Proceedings of the International Conference on High-Energy Nuclear Physics, Geneva, 1962* (CERN Scientific Information Service, Geneva, Switzerland, 1962), p. 303; more extensive analysis is presented in Ref. 1.

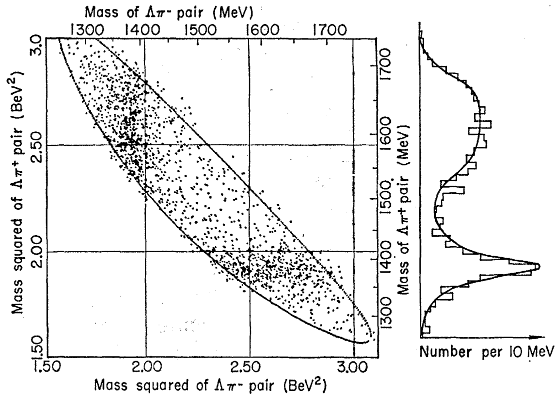

FIG. 1. Dalitz plot of $\Lambda\pi^+\pi^-$ events from $K^- - p$ interactions at 1.22 BeV/c. Projection of the events onto the $\Lambda\pi^+$ mass axis is displayed to the right of the figure; the curve represents the fitting of Breit-Wigner resonance expressions to the $\Lambda\pi^+$ and $\Lambda\pi^-$ systems.

The various t_L^M moments determined are presented in Table I. [The negative-M moments are omitted, as they give no additional information, t_L^{-M} being equal to $(-)^M t_L^{M*}$.] There are two evaluations for each odd-L moment, one from longitudinal and one from transverse polarization. These are compared below. The power series in sines and cosines necessary to describe the data are obtained by using the experimental moments of Table I in the expressions (1) through (4); e.g., the longitudinal polarization for the assumption of the Y_1^* state $P_{3/2}$ is given by

$$I\bar{P}\cdot\hat{\Lambda}=n_{10}t_1^0Y_1^0+n_{30}t_3^2Y_3^{2*}+n_{30}t_3^0Y_3^{0*}+n_{30}t_3^{-2}Y_3^{-2*} =(3/4\pi)^{1/2}[0.126\ t_1^0\cos\theta-0.379(35/2)^{1/2} \times(\mathrm{Re}t_3^2\sin^2\theta\cos\theta\cos2\phi+\mathrm{Im}t_3^2\sin^2\theta\cos\theta\sin2\phi) -0.379\ t_3^0(\sqrt{7}/2\sqrt{3})(5\cos^3\theta-3\cos\theta)]. \quad (16)$$

To decide the ***spin*** of the Y_1^*, the maximum complexity of nonzero moments was determined. A chi-squared was constructed of the form

$$\chi^2=\sum_{i,j}(t_i-\langle t_i\rangle)G_{ij}^{-1}(t_j-\langle t_j\rangle), \quad (17)$$

where t_i or t_j represents a real or imaginary part of any t_L^M moment (e.g., $(1/n_{20})\times\langle\mathrm{Re}Y_2^2\rangle$) and $\langle t_i\rangle$ or $\langle t_j\rangle$ designates the expected value; the moments included were those needed to describe the decay of a spin $\frac{5}{2}$ system (17 real numbers).[12] The G^{-1} is the inverse error matrix; each term of the error matrix is given by

$$G_{ij}=\langle\delta(t_i-\langle t_i\rangle)\delta(t_j-\langle t_j\rangle)\rangle =(1/N^2)\sum_{k=1}^{N}[Y_i(k)-\langle Y_i\rangle][Y_j(k)-\langle Y_j\rangle]. \quad (18)$$

[12] Moments higher than $L=5$ have not been examined; and a χ^2 for spin $\frac{5}{2}$ is not presented. From the fact that all 10 independent parameters for the $L=4$ and $L=5$ moments were consistent with zero and from the lack of any evidence for $(\hat{\Lambda}\cdot\hat{n})^6$ polarization terms in the earlier study, these are expected to be zero.

[For diagonal terms Eq. (18) is the same as Eq. (13) or Eq. (14).] Data from the angular distribution and from all polarization distributions were used.

The χ^2 for Y_1^* spin equal to $\frac{1}{2}$ was found by using for the expected $\langle t_i \rangle$ the experimental values for t_{10}, $\text{Re}t_{11}$, and $\text{Im}t_{11}$, but taking all higher-L $\langle t \rangle$'s to be zero (as required for spin $\frac{1}{2}$). The χ^2 for spin $\frac{3}{2}$ was obtained in a similar way. The results are stated, with the number of degrees of freedom, in Table IIA.[12]

To determine the *parity* of the Y_1^* for each spin hypothesis, it was necessary to test equality of the corresponding moments for longitudinal and transverse components of polarization with the γ of Eq. (12) assumed equal to $+1$ or -1. The χ^2 formed was

$$\chi^2 = \sum_{i,j} (t_i^L - \gamma t_i^T/\gamma') G_{ij}^{-1} (t_j^L - \gamma t_j^T/\gamma'), \qquad (19)$$

where the indices i and j again designate any $\text{Re}t_L^M$ or $\text{Im}t_L^M$ (including only those permitted to be nonzero for a given spin), and where superscripts L and T denote longitudinal or transverse moments [cf. Eq. (24)]. Each element of the error matrix has the form

$$G_{ij} = \langle \delta(t_i^L - \gamma t_i^T/\gamma') \delta(t_j^L - \gamma t_j^T/\gamma') \rangle. \qquad (20)$$

The constant γ' was given a value of $+1$ to test for $l = J - \frac{1}{2}$ and -1 for $l = J + \frac{1}{2}$. Results of the parity test are given in Table IIB.

Another method investigated for treating polarization data was a ratio techinique also advanced by Byers and Fenster.[13] A ratio of the experimental evaluations of the two parts of Eq. (12) (the A_L sum and the B_L sum) was compared with theoretically predicted ratios for various spin-parity hypotheses. The evaluations of a χ^2 [which tested (part A) = (part B) $\times R$, with R the predicted ratio] were identical with those of the parity χ^2 discussed above and presented in Table II; however, the results permitted no discrimination between spin $\frac{1}{2}$ and spin $\frac{3}{2}$.

Additional attempts were made to discriminate between spin-$\frac{3}{2}$ and spin-$\frac{5}{2}$ hypotheses. The experimental moments of Table I were used to evaluate

$$\sum_{L,M} (2L+1) |t_L^M|^2 \leqslant 2J+1 \qquad (21)$$

and

$$2 \sum_{L,M} (2L+1) |t_L^M|^2 \geqslant 2J+1, \qquad (22)$$

both of these deriving from general properties of the density matrix.[14] Both inequalities were found well satisfied, within errors, for $J = \frac{3}{2}$ and $J = \frac{5}{2}$.

Another study of spin hypotheses involved the explicit solution for spin J from the Byers-Fenster relation between longitudinal and transverse polarization moments.[6] The equality

$$t_L^M \text{ (longitudinal)} = t_L^M \text{ (transverse)} \qquad (23)$$

demands that

$$(1/n_{L0}^J) P_{||}^{(LM)} = (1/\gamma n_{L1}^J) P_{\perp}^{(LM)}, \qquad (24)$$

where $P_{||}^{(LM)}$ and $P_{\perp}^{(LM)}$ are the L, M moments of the distributions in Eqs. (3) and (4), and are evaluated according to Eqs. (11) and (12). As shown by Byers and Fenster, Eq. (24) is equivalent to

$$\gamma(2J+1) = [L(L+1)]^{1/2} P_{\perp}^{(LM)} / P_{||}^{(LM)} \qquad (25)$$

since $n_{L1}^J = (2J+1)[L(L+1)]^{-\frac{1}{2}} n_{L0}^J$. This equation should hold for every value of L and M for which a polarization moment can be defined. The use of the four lowest moments (proportional to t_1^0, $\text{Re}t_3^2$, $\text{Im}t_3^2$, and t_3^0) to evaluate J from Eq. (25) indicates that the spin is likely to be $\frac{3}{2}$ rather than $\frac{5}{2}$. (See Table III.) A simple χ^2 of the form

$$\chi^2 = \sum_i (J' - J_i)^2 / (\delta J_i)^2 \qquad (26)$$

yields confidence levels of 0.005 for $J' = \frac{5}{2}$ and 0.22 for $J' = \frac{3}{2}$. However, these χ^2 values cannot be considered reliable. {They differ substantially from those of the parity χ^2 [Eq. (19)], though based on the same

TABLE I. t_L^M moments.

J	L	M	$\text{Re}t_L^M$	$\text{Im}t_L^M$
		A. From $I(\theta,\phi)$		
$\frac{1}{2}, \frac{3}{2}, \frac{5}{2}$	0	0	1.0000	
$\frac{3}{2}$	2	+2	-0.018 ± 0.022	-0.028 ± 0.023
	2	0	-0.118 ± 0.034	
$\frac{5}{2}$	2	+2	-0.017 ± 0.021	-0.026 ± 0.021
	2	0	-0.110 ± 0.032	
	4	+4	-0.025 ± 0.024	-0.011 ± 0.024
	4	+2	0.014 ± 0.025	0.042 ± 0.025
	4	0	0.027 ± 0.037	
		B. From $I\bar{P}(\theta,\phi)$—longitudinal component		
J	L	M	$\text{Re}t_L^M$	$\text{Im}t_L^M$
$\frac{1}{2}$	1	0	-0.019 ± 0.100	
$\frac{3}{2}$	1	0	-0.043 ± 0.225	
	3	+2	-0.134 ± 0.048	-0.066 ± 0.051
	3	0	0.247 ± 0.073	
$\frac{5}{2}$	1	0	-0.066 ± 0.343	
	3	+2	-0.246 ± 0.088	-0.121 ± 0.094
	3	0	0.454 ± 0.134	
	5	+4	0.046 ± 0.042	0.025 ± 0.043
	5	+2	0.045 ± 0.044	0.008 ± 0.046
	5	0	0.030 ± 0.065	
		C. From $I\bar{P}(\theta,\phi)$—transverse component		
J	L	M	$\gamma\ \text{Re}t_L^M$	$\gamma\ \text{Im}t_L^M$
$\frac{1}{2}$	1	0	-0.051 ± 0.061	
$\frac{3}{2}$	1	0	-0.056 ± 0.068	
	3	+2	-0.077 ± 0.041	0.040 ± 0.041
	3	0	0.272 ± 0.058	
$\frac{5}{2}$	1	0	-0.057 ± 0.070	
	3	+2	-0.095 ± 0.050	0.048 ± 0.050
	3	0	0.332 ± 0.071	
	5	+4	0.026 ± 0.037	-0.029 ± 0.038
	5	+2	0.012 ± 0.038	-0.029 ± 0.037
	5	0	0.015 ± 0.055	

[13] N. Byers and S. Fenster (UCLA, unpublished department report) and (private communication).

[14] See Byers and Fenster, Ref. 6. See also R. H. Capps, Phys. Rev. **122**, 929 (1961) for discussion of Eq. (22), the Eberhard-Good theorem.

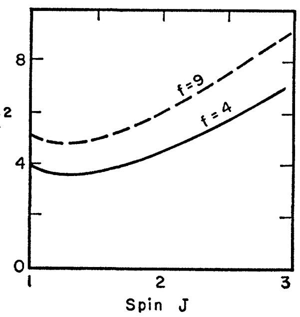

FIG. 2. Dependence of the "parity χ^2" [Eq. (19)] on spin J. The solid line represents the χ^2 obtained from the four moments ($L=1$, 3) appropriate to spin $\frac{3}{2}$; the dashed line represents the χ^2 evaluated for the nine moments ($L=1$, 3, 5) appropriate to spin $\frac{5}{2}$. The number of degrees of freedom f is 4 in the former case and 9 in the latter.

$P_{||}^{(LM)}-P_{\perp}^{(LM)}$ relation, Eq. (24); the differences are much too great to be accounted for by the neglect of correlated errors in Eq. (26).} The experimental values J_i of Eq. (26) are not Gaussianly distributed, as they depend on the *ratio* of $P_{\perp}^{(i)}$ to $P_{||}^{(i)}$. The "parity χ^2," however, does test a Gaussianly-distributed quantity, $(1/n_{L0})P_{||}^{(i)}-(1/\gamma n_{L1})P_{\perp}^{(i)}$.

The value of J was varied in small increments in the "parity χ^2," with first the four moments appropriate to spin $\frac{3}{2}$ and then the nine moments appropriate to spin $\frac{5}{2}$. Only a slow rise in each χ^2 is observed as J is increased from $\frac{3}{2}$ to $\frac{5}{2}$; the confidence levels for the four-moment χ^2 are 0.45 and 0.21, respectively. (See Fig. 2.) It is evident from the form of the nine-moment χ^2 as a function of J, that the inclusion of the $L=5$ moments has no significant effect on the χ^2 (as might be expected from the fact that these had near-zero values); the χ^2 minimum is still near $J=\frac{3}{2}$. These results represent the best reliable discrimination between $J=\frac{3}{2}$ and $J=\frac{5}{2}$ obtained from the data.

V. COMPARISON WITH EARLIER ANALYSIS

The coefficients or moments obtained in the study presented here were checked by comparing distributions

TABLE II. χ^2 values.

A. Y_1^* spin

Y_1^* spin	χ^2	Degrees of freedom	Confidence level
$\frac{1}{2}$	77.0	24	2×10^{-7}
$\frac{3}{2}$	10.8	15	0.79

B. Y_1^* parity

Y_1^* state	Parity	χ^2	Degrees of freedom	Confidence level
$S_{1/2}$	(−)	0.07	1	0.80
$P_{1/2}$	(+)	0.34	1	0.56
$P_{3/2}$	(+)	3.7	4	0.45
$D_{3/2}$	(−)	44.9	4	$<10^{-7}$
$D_{5/2}$	(−)	7.6	9	0.57
$F_{5/2}$	(+)	45.3	9	8×10^{-7}

TABLE III. J values from Eq. (25).

Moment:	t_1^0	$\mathrm{Re}t_3^2$	$\mathrm{Im}t_3^2$	t_3^0
J:	2.1 ± 14.1	0.65 ± 0.73	-1.7 ± 1.6	1.7 ± 0.8

obtained with the functions of lesser complexity that the authors previously derived and fitted to the data. The distributions given in Eqs. (1) through (4), with experimental t_L^M substituted, were averaged over the azimuthal angle ϕ and then compared with the angular distribution and polarization distributions of the earlier Y_1^* report.[1] [$IP_{||}$ was compared with $(N\bar{P}\cdot\hat{n}+N\bar{P}\cdot\hat{m})/2\cos\theta$, and $\mathrm{Re}IP_{\perp}$ was compared with $(N\bar{P}\cdot\hat{m}-N\bar{P}\cdot\hat{n})/2\sin\theta$.] The relative magnitudes of coefficients and their errors compared very well.

VI. CONCLUSIONS

The assignment of spin $\frac{1}{2}$ to the Y_1^* is excluded by the existence of angular distribution and polarization moments of higher order ($L=2$, 3) than permitted. Spin $\frac{3}{2}$ seems quite acceptable, since moments expected for spin $\frac{5}{2}$ are consistent with zero. Spin $\frac{5}{2}$ is not required by the data. (See Table II for χ^2 values.) Several evaluations of the quantity $2J+1$ indicate that the Y_1^* spin J is more likely $\frac{3}{2}$ than $\frac{5}{2}$. (See Table III.)

The parity assignment demanded by longitudinal and transverse polarization moments is $\gamma=+1$ or $P_{3/2}$ for spin $\frac{3}{2}$. This confirms the earlier report.[1]

A further study now in progress treats the Y_1^* data with a maximum likelihood approach to find moments. Preliminary results on data ranging from 1.15 to 1.30 BeV/c are similar to the results given above.

A qualifying remark should be made with respect to confidence levels stated in Table II. As is no doubt obvious to the informed reader, quantitative values can be changed by a small amount of background. Thus the 10^{-6} or 10^{-7} confidence levels should not be taken too literally; but conclusions should be weighed appropriately with other pieces of evidence (which also have statistical and background uncertainties). It is reassuring that several experiments showing strong effects give consistent results for the Y_1^*. Perhaps the application of such techniques as the moment analysis to Y_1^* events in new K^- and π experiments will permit firm discrimination between $P_{3/2}$ and $D_{5/2}$ hypotheses.[15]

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[15] *Note added in proof.* The analysis described above has been carried out with 3600 additional Y_1^* events at 6 other incident K^- momentum settings. In 2 of these 6 studies, the confidence limit for $J=\frac{5}{2}$ [with the legitimate test of Eq. (24)] was $\leq1\%$.